MINISTRY OF EDUCATION, SINGAPORE in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME


CENTRE NUMBER


INDEX NUMBER


## ADDITIONAL MATHEMATICS

Paper 2
For examination from 2021

## SPECIMEN PAPER

2 hours 15 minutes
Candidates answer on the Question Paper.
No Additional Materials are required.

## READ THESE INSTRUCTIONS FIRST

Write your centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE ON ANY BARCODES.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90 .

This document consists of 19 printed pages and 1 blank page.

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Binomial expansion

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n},
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1) \ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A \\
\sin (A \pm B)=\sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B)=\cos A \cos B \mp \sin A \sin B \\
\tan (A \pm B)=\frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2 A=2 \sin A \cos A \\
\cos 2 A=\cos ^{2} A-\sin ^{2} A=2 \cos ^{2} A-1=1-2 \sin ^{2} A \\
\tan 2 A=\frac{2 \tan A}{1-\tan ^{2} A}
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1 Express $\frac{2 x^{3}-8}{x^{3}+4 x}$ in partial fractions.

2 (a) Variables $x$ and $y$ are related by the equation $y x^{n}=k$, where $n$ and $k$ are constants. Explain clearly how $n$ and $k$ can be calculated when a graph of $\lg y$ against $\lg x$ is drawn.
(b) The time for a complete oscillation, $t$ seconds, of a pendulum of length $l \mathrm{~m}$ is proportional to $\sqrt{l}$. In an experiment with pendulums of different lengths, the following table was obtained.

| Length of <br> pendulum, $l \mathrm{~m}$ | 0.2 | 0.4 | 0.6 | 1.0 |
| :--- | :---: | :---: | :---: | :---: |
| Time of one <br> oscillation, $t \mathrm{sec}$ | 0.90 | 1.27 | 1.55 | 2.02 |

(i) On the grid on page 5 , draw a straight line graph to illustrate this data.
(ii) Use your graph to estimate the time of one oscillation for a pendulum of length 0.8 m .

It is known that the correct formula connecting $t$ and $l$ is $t=2 \pi \sqrt{\frac{l}{g}}$, where $g$ is the acceleration due to gravity.
(iii) Use your graph to estimate the value for $g$.



The diagram shows part of the curve $y=2 x-\frac{16}{x^{2}}$, which intersects the $x$-axis at $P$. The tangent to the curve at $P$ meets the line $x=4$ at $Q$.
(a) Find the equation of the line $P Q$.
(b) Find the area of the shaded region bounded by the tangent $P Q$, the curve and the line $x=4$.

4 (a) A curve has equation $y=\frac{2}{x}+k$ and a line has equation $2 x+3 y=k$, where $k$ is a constant. Find the set of values of $k$ for which the curve and the line do not intersect and represent this set on a number line.
(b) The curve with equation $y=a x^{2}+b x+a$, where $a$ and $b$ are constants, lies completely below the $x$-axis.
(i) Write down the conditions which must apply to $a$ and $b$.
(ii) Give an example of possible values for $a$ and $b$ which satisfy the conditions in part (i).

5 (a) Express $\frac{2 x}{2 x+3}$ in the form $a+\frac{b}{2 x+3}$ where $a$ and $b$ are constants, and hence find $\int \frac{2 x}{2 x+3} \mathrm{~d} x$.
(b) Given that $y=x \ln (2 x+3)$, find an expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(c) Using the results from parts (a) and (b), find $\int \ln (2 x+3) \mathrm{d} x$.


The diagram shows the path taken by a lady hiker. She walks 8 km in a straight line from a point $A$ to a point $B$. The path $A B$ is inclined at an acute angle $\theta^{\circ}$ to a straight road $A D$. Having reached $B$, she turns through $90^{\circ}$ and walks 5 km to a point $C$, finding herself $x \mathrm{~km}$ from the road.
(a) Show that $x=8 \sin \theta-5 \cos \theta$.
(b) Express $8 \sin \theta-5 \cos \theta$ in the form $R \sin (\theta-\alpha)$, where $R>0$ and $0^{\circ} \leqslant \alpha \leqslant 90^{\circ}$.
(c) Given that $x=2$, find the value of $\theta$.

7 The equation of a curve is $y=\mathrm{e}^{x^{2}-4 x}$.
(a) Find expressions for $\frac{\mathrm{d} y}{\mathrm{~d} x}$ and $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$.
(b) Find the exact value of the coordinates of the stationary point.
(c) Find the nature of the stationary point.

8 The points $P$ and $Q$ both lie on a circle and have coordinates $(-2,7)$ and $(6,1)$ respectively. The centre of the circle lies on the line $y=x+4$.
(a) Find the equation of the perpendicular bisector of $P Q$.
(b) Find the equation of the circle.

The point $R$ is such that $P R$ is a diameter of the circle.
(c) Find the coordinates of $R$.

9 (a) Use the substitution $u=2^{x}$ to solve the equation $2^{2 x}-2^{x+2}=5$.
(b) The equation $\log _{2} x+\log _{8} x=\log _{5} 25$ has the solution $x=2^{k}$. Find the value of $k$.
(c) Show that the equation $\log _{3}(4 x-11)-\log _{3}(x-3)=1$ has no real solutions.

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