Mathematics

Singapore-Cambridge General Certificate of Education Ordinary Level (2020)

(Syllabus 4048)

CONTENTS

	Page
INTRODUCTION	**2**
AIMS	**2**
ASSESSMENT OBJECTIVES	**2**
SCHEME OF ASSESSMENT	**3**
USE OF CALCULATORS	**3**
SUBJECT CONTENT	**4**
MATHEMATICAL FORMULAE	**11**
MATHEMATICAL NOTATION	**12**

The Common Last Topics highlighted in yellow will not be examined in 2020 O-Level national examination.

Singapore Examinations and Assessment Board

CAMBRIDGE International Examinations

© MOE & UCLES 2018

INTRODUCTION

The syllabus is intended to provide students with the fundamental mathematical knowledge and skills. The content is organised into three strands namely, *Number and Algebra*, *Geometry and Measurement*, and *Statistics and Probability*. Besides conceptual understanding and skills proficiency explicated in the content strands, development of process skills that are involved in the process of acquiring and applying mathematical knowledge is also emphasised. These include reasoning, communication and connections, thinking skills and heuristics, and application and modelling; and are developed based on the three content strands.

AIMS

The O-Level Mathematics syllabus aims to enable all students to:

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving
- connect ideas within mathematics and between mathematics and other subjects through applications of mathematics
- build confidence and foster interest in mathematics.

ASSESSMENT OBJECTIVES

The assessment will test candidates' abilities to:

AO1 understand and apply mathematical concepts and skills in a variety of contexts

AO2 organise and analyse data and information; formulate and solve problems, including those in real-world contexts, by selecting and applying appropriate techniques of solution; interpret mathematical results

AO3 solve higher order thinking problems; make inferences; write mathematical explanation and arguments.

SCHEME OF ASSESSMENT

Paper	Duration	Description	Marks	Weighting
Paper 1	2 hours	There will be about 25 short answer questions. Candidates are required to answer **all** questions.	80	50%
Paper 2	2 hours 30 minutes	There will be 10 to 11 questions of varying marks and lengths. The last question in this paper will focus specifically on applying mathematics to a real-world scenario. Candidates are required to answer **all** questions.	100	50%

NOTES

1. Omission of essential working will result in loss of marks.

2. Some questions may integrate ideas from more than one topic of the syllabus where applicable.

3. Relevant mathematical formulae will be provided for candidates.

4. Candidates should have geometrical instruments with them for Paper 1 and Paper 2.

5. Unless stated otherwise within a question, three-figure accuracy will be required for answers. This means that four-figure accuracy should be shown throughout the working, including cases where answers are used in subsequent parts of the question. Premature approximation will be penalised, where appropriate. Angles in degrees should be given to one decimal place.

6. SI units will be used in questions involving mass and measures.
   Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15; 3.15 p.m. by 15 15.

7. Candidates are expected to be familiar with the solidus notation for the expression of compound units, e.g. 5 cm/s for 5 centimetres per second, 13.6 g/cm^3 for 13.6 grams per cubic centimetre.

8. Unless the question requires the answer in terms of π, the calculator value for π or π = 3.142 should be used.

9. Spaces will be provided in each question paper for working and answers.

USE OF CALCULATORS

An approved calculator may be used in **both** Paper 1 and Paper 2.

SUBJECT CONTENT

Topic/Sub-topics		Content
NUMBER AND ALGEBRA		
N1	**Numbers and their operations**	• primes and prime factorisation • finding highest common factor (HCF) and lowest common multiple (LCM), squares, cubes, square roots and cube roots by prime factorisation • negative numbers, integers, rational numbers, real numbers, and their four operations • calculations with calculator • representation and ordering of numbers on the number line • use of the symbols $<$, $>$, $\leqslant$, $\geqslant$ • approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures and estimating the results of computation) • use of standard form $A \times 10^n$, where n is an integer, and $1 \leqslant A < 10$ • positive, negative, zero and fractional indices • laws of indices
N2	**Ratio and proportion**	• ratios involving rational numbers • writing a ratio in its simplest form • map scales (distance and area) • direct and inverse proportion
N3	**Percentage**	• expressing one quantity as a percentage of another • comparing two quantities by percentage • percentages greater than 100% • increasing/decreasing a quantity by a given percentage • reverse percentages
N4	**Rate and speed**	• average rate and average speed • conversion of units (e.g. km/h to m/s)

Topic/Sub-topics		Content
N5	**Algebraic expressions and formulae**	• using letters to represent numbers • interpreting notations: ∗ ab as $a \times b$ ∗ $\frac{a}{b}$ as $a \div b$ or $a \times \frac{1}{b}$ ∗ a^2 as $a \times a$, a^3 as $a \times a \times a$, a^2b as $a \times a \times b$, … ∗ $3y$ as $y + y + y$ or $3 \times y$ ∗ $3(x + y)$ as $3 \times (x + y)$ ∗ $\frac{3+y}{5}$ as $(3 + y) \div 5$ or $\frac{1}{5} \times (3 + y)$ • evaluation of algebraic expressions and formulae • translation of simple real-world situations into algebraic expressions • recognising and representing patterns/relationships by finding an algebraic expression for the nth term • addition and subtraction of linear expressions • simplification of linear expressions such as: $-2(3x - 5) + 4x$ $\frac{2x}{3} - \frac{3(x-5)}{2}$ • use brackets and extract common factors • factorisation of linear expressions of the form $ax + bx + kay + kby$ • expansion of the product of algebraic expressions • changing the subject of a formula • finding the value of an unknown quantity in a given formula • use of: ∗ $(a + b)^2 = a^2 + 2ab + b^2$ ∗ $(a - b)^2 = a^2 - 2ab + b^2$ ∗ $a^2 - b^2 = (a + b)(a - b)$ • factorisation of quadratic expressions $ax^2 + bx + c$ • multiplication and division of simple algebraic fractions such as: $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ $\frac{3a}{4} \div \frac{9a^2}{10}$ • addition and subtraction of algebraic fractions with linear or quadratic denominator such as: $\frac{1}{x-2} + \frac{2}{x-3}$ $\frac{1}{x^2-9} + \frac{2}{x-3}$ $\frac{1}{x-3} + \frac{2}{(x-3)^2}$

Topic/Sub-topics		Content
N6	**Functions and graphs**	• Cartesian coordinates in two dimensions • graph of a set of ordered pairs as a representation of a relationship between two variables • linear functions ($y = ax + b$) and quadratic functions ($y = ax^2 + bx + c$) • graphs of linear functions • the gradient of a linear graph as the ratio of the vertical change to the horizontal change (positive and negative gradients) • graphs of quadratic functions and their properties: ∗ positive or negative coefficient of x^2 ∗ maximum and minimum points ∗ symmetry • sketching the graphs of quadratic functions given in the form: ∗ $y = (x - p)^2 + q$ ∗ $y = -(x - p)^2 + q$ ∗ $y = (x - a)(x - b)$ ∗ $y = -(x - a)(x - b)$ • graphs of power functions of the form $y = ax^n$, where $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these • graphs of exponential functions $y = ka^x$, where a is a positive integer • estimation of the gradient of a curve by drawing a tangent
N7	**Equations and inequalities**	• solving linear equations in one variable • solving simple fractional equations that can be reduced to linear equations such as: $\frac{x}{3} + \frac{x-2}{4} = 3$ $\frac{3}{x-2} = 6$ • solving simultaneous linear equations in two variables by ∗ substitution and elimination methods ∗ graphical method • solving quadratic equations in one unknown by ∗ factorisation ∗ use of formula ∗ completing the square for $y = x^2 + px + q$ ∗ graphical methods • solving fractional equations that can be reduced to quadratic equations such as: $\frac{6}{x+4} = x + 3$ $\frac{1}{x-2} + \frac{2}{x-3} = 5$ • formulating equations to solve problems • solving linear inequalities in one variable, and representing the solution on the number line

Topic/Sub-topics		Content
N8	**Set language and notation**	• use of set language and the following notation: Union of A and B — $A \cup B$ Intersection of A and B — $A \cap B$ '… is an element of …' — $\in$ '… is not an element of …' — $\notin$ Complement of set A — A' The empty set — $\varnothing$ Universal set — $\mathscr{E}$ A is a (proper) subset of B — $A \subset B$ A is not a (proper) subset of B — $A \not\subset B$ • union and intersection of two sets • Venn diagrams
N9	**Matrices**	• display of information in the form of a matrix of any order • interpreting the data in a given matrix • product of a scalar quantity and a matrix • problems involving the calculation of the sum and product (where appropriate) of two matrices
N10	**Problems in real-world contexts**	• solving problems based on real-world contexts: ∗ in everyday life (including travel plans, transport schedules, sports and games, recipes, etc.) ∗ involving personal and household finance (including simple and compound interest, taxation, instalments, utilities bills, money exchange, etc.) • interpreting and analysing data from tables and graphs, including distance–time and speed–time graphs • interpreting the solution in the context of the problem
GEOMETRY AND MEASUREMENT		
G1	**Angles, triangles and polygons**	• right, acute, obtuse and reflex angles • vertically opposite angles, angles on a straight line and angles at a point • angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles • properties of triangles, special quadrilaterals and regular polygons (pentagon, hexagon, octagon and decagon), including symmetry properties • classifying special quadrilaterals on the basis of their properties • angle sum of interior and exterior angles of any convex polygon • properties of perpendicular bisectors of line segments and angle bisectors • construction of simple geometrical figures from given data (including perpendicular bisectors and angle bisectors) using compasses, ruler, set squares and protractors, where appropriate

Topic/Sub-topics		Content
G2	**Congruence and similarity**	• congruent figures and similar figures • properties of similar triangles and polygons: ∗ corresponding angles are equal ∗ corresponding sides are proportional • enlargement and reduction of a plane figure • scale drawings • determining whether two triangles are ∗ congruent ∗ similar • ratio of areas of similar plane figures • ratio of volumes of similar solids • solving simple problems involving similarity and congruence
G3	**Properties of circles**	• symmetry properties of circles: ∗ equal chords are equidistant from the centre ∗ the perpendicular bisector of a chord passes through the centre ∗ tangents from an external point are equal in length ∗ the line joining an external point to the centre of the circle bisects the angle between the tangents • angle properties of circles: ∗ angle in a semicircle is a right angle ∗ angle between tangent and radius of a circle is a right angle ∗ angle at the centre is twice the angle at the circumference ∗ angles in the same segment are equal ∗ angles in opposite segments are supplementary
G4	**Pythagoras' theorem and trigonometry**	• use of Pythagoras' theorem • determining whether a triangle is right-angled given the lengths of three sides • use of trigonometric ratios (sine, cosine and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles • extending sine and cosine to obtuse angles • use of the formula $\frac{1}{2}ab\sin C$ for the area of a triangle • use of sine rule and cosine rule for any triangle • problems in two and three dimensions including those involving angles of elevation and depression and bearings
G5	**Mensuration**	• area of parallelogram and trapezium • problems involving perimeter and area of composite plane figures • volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere • conversion between cm^2 and m^2, and between cm^3 and m^3 • problems involving volume and surface area of composite solids • arc length, sector area and area of a segment of a circle • use of radian measure of angle (including conversion between radians and degrees)

Topic/Sub-topics		Content
G6	**Coordinate geometry**	• finding the gradient of a straight line given the coordinates of two points on it • finding the length of a line segment given the coordinates of its end points • interpreting and finding the equation of a straight line graph in the form $y = mx + c$ • geometric problems involving the use of coordinates
G7	**Vectors in two dimensions**	• use of notations: $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$, $\mathbf{a}$, $\left\lvert\overrightarrow{AB}\right\rvert$ and $\lvert\mathbf{a}\rvert$ • representing a vector as a directed line segment • translation by a vector • position vectors • magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ • use of sum and difference of two vectors to express given vectors in terms of two coplanar vectors • multiplication of a vector by a scalar • geometric problems involving the use of vectors
G8	**Problems in real-world contexts**	• solving problems in real-world contexts (including floor plans, surveying, navigation, etc.) using geometry • interpreting the solution in the context of the problem

Topic/Sub-topics		Content
STATISTICS AND PROBABILITY		
S1	**Data analysis**	• analysis and interpretation of: ∗ tables ∗ bar graphs ∗ pictograms ∗ line graphs ∗ pie charts ∗ dot diagrams ∗ histograms with equal class intervals ∗ stem-and-leaf diagrams ∗ cumulative frequency diagrams ∗ box-and-whisker plots • purposes and uses, advantages and disadvantages of the different forms of statistical representations • explaining why a given statistical diagram leads to misinterpretation of data • mean, mode and median as measures of central tendency for a set of data • purposes and use of mean, mode and median • calculation of the mean for grouped data • quartiles and percentiles • range, interquartile range and standard deviation as measures of spread for a set of data • calculation of the standard deviation for a set of data (grouped and ungrouped) • using the mean and standard deviation to compare two sets of data
S2	**Probability**	• probability as a measure of chance • probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability) • probability of simple combined events (including using possibility diagrams and tree diagrams, where appropriate) • addition and multiplication of probabilities (mutually exclusive events and independent events)

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, {0, ±1, ±2, ±3, …}
$\mathbb{Z}^+$	the set of positive integers, {1, 2, 3, …}
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real *n*-tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\not\subseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\not>$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\not<$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b, ab, a.b$	a multiplied by b
$a \div b, \frac{a}{b}, a/b$	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$
	$\frac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	the function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx ; δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \, \mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}$, $\ddot{x}$, …	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

$\sin$, $\cos$, $\tan$, cosec, $\sec$, $\cot$	} the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	} the inverse circular functions

7. *Complex Numbers*

i	the square root of –1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\mathrm{Re}\, z$	the real part of z, $\mathrm{Re}\,(x + \mathrm{i}y) = x$
$\mathrm{Im}\, z$	the imaginary part of z, $\mathrm{Im}\,(x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{x^2 + y^2}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
i, j, k	unit vectors in the directions of the Cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations, $x_1, x_2, \ldots$ occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
$\mathrm{f}(x), \mathrm{g}(x) \ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x), \mathrm{G}(x) \ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binomial distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum(x - \bar{x})^2$
φ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample