Additional Mathematics

Singapore-Cambridge General Certificate of Education Ordinary Level (2020)

(Syllabus 4047)

CONTENTS

The Common Last Topics highlighted in yellow will not be examined in 2020 O-Level national examination.

Singapore Examinations and Assessment Board

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INTRODUCTION

The syllabus prepares students adequately for A-Level H2 Mathematics, where a strong foundation in algebraic manipulation skills and mathematical reasoning skills are required. The content is organised into three strands, namely, *Algebra, Geometry and Trigonometry*, and *Calculus*. Besides conceptual understanding and skill proficiency explicated in the content strand, the development of process skills, namely, reasoning, communication and connections, thinking skills and heuristics, and applications and modelling are also emphasised. The O-Level Additional Mathematics syllabus assumes knowledge of O-Level Mathematics.

AIMS

The O-Level Additional Mathematics syllabus aims to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, in particular, the sciences
- develop thinking, reasoning and metacognitive skills through a mathematical approach to problem-solving
- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics
- appreciate the abstract nature and power of mathematics.

ASSESSMENT OBJECTIVES

The assessment will test candidates' abilities to:

AO1 understand and apply mathematical concepts and skills in a variety of contexts

AO2 analyse information; formulate and solve problems, including those in real-world contexts, by selecting and applying appropriate techniques of solution; interpret mathematical results

AO3 solve higher order thinking problems; make inferences; reason and communicate mathematically through writing mathematical explanation, arguments and proofs.

SCHEME OF ASSESSMENT

Paper	Duration	Description	Marks	Weighting
Paper 1	2 h	There will be 11–13 questions of varying marks and lengths. Candidates are required to answer **ALL** questions.	80	44%
Paper 2	$2\frac{1}{2}$ h	There will be 9–11 questions of varying marks and lengths. Candidates are required to answer **ALL** questions.	100	56%

NOTES

1. Omission of essential working will result in loss of marks.

2. Some questions may integrate ideas from more than one topic of the syllabus where applicable.

3. Relevant mathematical formulae will be provided for candidates.

4. Unless stated otherwise within a question, three-figure accuracy will be required for answers. Angles in degrees should be given to one decimal place.

5. SI units will be used in questions involving mass and measures.
   Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15; 3.15 p.m. by 15 15.

6. Candidates are expected to be familiar with the solidus notation for the expression of compound units, e.g. 5 m/s for 5 metres per second.

7. Unless the question requires the answer in terms of π, the calculator value for π or π = 3.142 should be used.

USE OF CALCULATORS

An approved calculator may be used in **both** Paper 1 and Paper 2.

SUBJECT CONTENT

Knowledge of the content of O-Level Mathematics syllabus is assumed in the syllabus below and will not be tested directly, but it may be required indirectly in response to questions on other topics.

<table>
<tr><th colspan="2">Topic/Sub-topics</th><th>Content</th></tr>
<tr><td colspan="3">ALGEBRA</td></tr>
<tr><td>A1</td><td>Equations and inequalities</td><td>• Conditions for a quadratic equation to have:
(i) two real roots
(ii) two equal roots
(iii) no real roots
and related conditions for a given line to:
(i) intersect a given curve
(ii) be a tangent to a given curve
(iii) not intersect a given curve
• Conditions for $ax^2 + bx + c$ to be always positive (or always negative)
• Solving simultaneous equations in two variables with at least one linear equation, by substitution
• Relationships between the roots and coefficients of a quadratic equation
• Solving quadratic inequalities, and representing the solution on the number line</td></tr>
<tr><td>A2</td><td>Indices and surds</td><td>• Four operations on indices and surds, including rationalising the denominator
• Solving equations involving indices and surds</td></tr>
<tr><td>A3</td><td>Polynomials and Partial Fractions</td><td>• Multiplication and division of polynomials
• Use of remainder and factor theorems
• Factorisation of polynomials
• Use of:
– $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$
– $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$
• Solving cubic equations
• Partial fractions with cases where the denominator is no more complicated than:
– $(ax + b)(cx + d)$
– $(ax + b)(cx + d)^2$
– $(ax + b)(x^2 + c^2)$</td></tr>
<tr><td>A4</td><td>Binomial expansions</td><td>• Use of the Binomial Theorem for positive integer n
• Use of the notations $n!$ and $\binom{n}{r}$
• Use of the general term $\binom{n}{r}a^{n-r}b^r$, $0 < r \leqslant n$ (knowledge of the greatest term and properties of the coefficients is not required)</td></tr>
</table>

Topic/Sub-topics		Content
A5	Power, Exponential, Logarithmic, and Modulus functions	• Power functions $y = ax^n$ where n is a simple rational number, and their graphs • Exponential and logarithmic functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs, including: – laws of logarithms – equivalence of $y = a^x$ and $x = \log_a y$ – change of base of logarithms • Modulus functions $\|x\|$ and $\|f(x)\|$ where $f(x)$ is linear, quadratic or trigonometric, and their graphs • Solving simple equations involving exponential, logarithmic and modulus functions
GEOMETRY AND TRIGONOMETRY		
G1	Trigonometric functions, identities and equations	• Six trigonometric functions for angles of any magnitude (in degrees or radians) • Principal values of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ • Exact values of the trigonometric functions for special angles (30°, 45°, 60°) or $\left(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$ • Amplitude, periodicity and symmetries related to the sine and cosine functions • Graphs of $y = a \sin (bx) + c$, $y = a \sin \left(\frac{x}{b}\right) + c$, $y = a \cos (bx) + c$, $y = a \cos \left(\frac{x}{b}\right) + c$ and $y = a \tan (bx)$, where a is real, b is a positive integer and c is an integer. • Use of the following – $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$, $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the expression for $a \cos\theta + b \sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R \sin(\theta \pm \alpha)$ • Simplification of trigonometric expressions • Solution of simple trigonometric equations in a given interval (excluding general solution) • Proofs of simple trigonometric identities

Topic/Sub-topics		Content
G2	Coordinate geometry in two dimensions	• Condition for two lines to be parallel or perpendicular • Midpoint of line segment • Area of rectilinear figure • Graphs of parabolas with equations in the form $y^2 = kx$ • Coordinate geometry of circles in the form: – $(x - a)^2 + (y - b)^2 = r^2$ – $x^2 + y^2 + 2gx + 2fy + c = 0$ (excluding problems involving 2 circles) • Transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from a straight line graph
G3	Proofs in plane geometry	• Use of: – properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles♦ – congruent and similar triangles♦ – midpoint theorem – tangent-chord theorem (alternate segment theorem)
Calculus		
C1	Differentiation and integration	• Derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a point • Derivative as rate of change • Use of standard notations $\mathrm{f}'(x), \mathrm{f}''(x), \frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left[= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right]$ • Derivatives of x^n, for any rational n, sin x, cos x, tan x, e^x, and ln x, together with constant multiples, sums and differences • Derivatives of products and quotients of functions • Derivatives of composite functions • Increasing and decreasing functions • Stationary points (maximum and minimum turning points and stationary points of inflexion) • Use of second derivative test to discriminate between maxima and minima • Applying differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems • Integration as the reverse of differentiation • Integration of x^n, for any rational n, sin x, cos x, $\sec^2 x$ and e^x, together with constant multiples, sums and differences • Integration of $(ax + b)^n$, for any rational n, $\sin(ax + b)$, $\cos(ax + b)$, and e^{ax+b} • Definite integral as area under a curve • Evaluation of definite integrals • Finding the area of a region bounded by a curve and line(s) (excluding area of region between two curves) • Finding areas of regions below the x-axis • Application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line

♦ These are properties learnt in O Level Mathematics.

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\ngtr$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\nless$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\dfrac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\dbinom{n}{r}$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$ $\dfrac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx ; δx	an increment of x
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y\,\mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y\,\mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}$, $\ddot{x}$, …	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions

7. *Complex Numbers*

i	square root of -1
z	a complex number, $z = x + \mathrm{i}y$
	$= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$
	$= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\operatorname{Re} z$	the real part of z, $\operatorname{Re}(x + \mathrm{i}y) = x$
$\operatorname{Im} z$	the imaginary part of z, $\operatorname{Im}(x + \mathrm{i}y) = y$
$\lvert z\rvert$	the modulus of z, $\lvert x + \mathrm{i}y\rvert = \sqrt{x^2 + y^2}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta)\rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert\mathbf{a}\rvert$	the magnitude of $\mathbf{a}$
$\lvert\overrightarrow{AB}\rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a}\times\mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations, x_1, x_2 …occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2 \cdots$	probabilities of the values $x_1, x_2, \ldots$of the discrete random variable X
$\mathrm{f}(x)$, $\mathrm{g}(x)\ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x)\ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binominal distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\Sigma(x-\bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample