

MINISTRY OF EDUCATION, SINGAPORE in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level
Higher 1


## MATHEMATICS

8865/01
Paper 1
For examination from 2025
SPECIMEN PAPER

## 3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

## READ THESE INSTRUCTIONS FIRST

## Answer all questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.
The number of marks is given in brackets [ ] at the end of each question or part question.

## Section A: Pure Mathematics [40 marks]

1 The curve $C$ has equation $y=1-2 \mathrm{e}^{-2 x}$. Without using a calculator, find the equation of the tangent to $C$ at the point where $x=\frac{1}{2}$, giving your answer in the form $y=m x+c$, where $m$ and $c$ are exact constants.

2 (a) Differentiate $3 \ln \left(2+5 x^{3}\right)$ with respect to $x$.
(b) Find $\int\left(2 \sqrt{x}-\frac{3}{x^{2}}\right)^{2} d x$.

3 A company is replacing some chairs, some desks and some tables in its offices. The manager knows the following information about costs.

- The total cost of 36 chairs, 25 desks and 9 tables is $\$ 7309$.
- The cost of 11 chairs is $\$ 576$ less than the cost of 4 tables.
- The cost of 14 desks is $\$ 38$ more than the total cost of 3 chairs and 3 tables.
(a) Express this information as three linear equations and hence find the cost of 1 desk.

A year later, the manager wishes to replace more chairs and desks. He finds that all the prices have increased by $5 \%$. He has a maximum of $\$ 4000$ to spend and he buys an equal number of chairs and desks.
(b) Find the greatest possible number of chairs that he can buy.

4 (a) Sketch the graph of the curve $C$ with equation $y=(x+2)(4-x)$.

The line $y=x+k$ intersects $C$ at two distinct points.
(b) Find the set of possible values of $k$.
(c) Without using a calculator, find the area of the region between $C$ and the line $y=x+6$.

$A B C D$ is a rectangle in which $A B=y \mathrm{~cm}$ and $B C=4 x \mathrm{~cm} . E F G H$ is a square of side $x \mathrm{~cm}$ such that $H G$ lies on $D C$ and the square is inside the rectangle (see diagram).

The perimeter of the rectangle $A B C D$ is 58 cm and the area of the trapezium $A B F E$ is $90 \mathrm{~cm}^{2}$.
Find the value of $x$ and the value of $y$.

6 A company manufactures televisions. The rate, $C$ thousand dollars per year, at which the total manufacturing costs change is to be monitored regularly over a period of 4 years. The company economist believes that the connection between $C$ and the time, $t$ years, can be modelled by the equation

$$
C=30-28 t+10 t^{2}-t^{3}, \quad \text { for } 0 \leqslant t \leqslant 4 .
$$

(a) Use differentiation and this model to find the minimum value of $C$, justifying that this value is a minimum.

The annual profit from the sale of these televisions is $P$ thousand dollars per year. The economist believes that the connection between $P$ and $t$ is given by

$$
P=2+18 \ln (t+1), \quad \text { for } 0 \leqslant t \leqslant 6 .
$$

(b) Find the exact value of $t$ for which $P=11$.
(c) Sketch the graph of $P$ against $t$.
(d) Deduce whether the annual profit is increasing or decreasing when $t=3$.

## Section B: Probability and Statistics [60 marks]

7 A new reading club has 15 members of whom 6 are men and 9 are women. A committee of 5 people is to be chosen from the 15 members.
(a) In how many ways can the committee be chosen if it must contain 2 men and 3 women?
(b) In how many ways can the committee be chosen if it must contain at least 3 women?

Suky and Tammy are two of the 9 women members of the club.
(c) In how many ways can the 9 women be arranged in a line with Suky and Tammy not standing next to each other?

8 A survey was conducted with a large number of small business owners who were asked to identify one business skill that they would like to develop further. The survey showed that $60 \%$ would like to develop skills in customer service, $30 \%$ would like to develop skills in marketing and $10 \%$ would like to develop skills in financial management.

Six business owners who took part in the survey were randomly selected.
(a) Using the binomial distribution, find the probability that,
(i) exactly four business owners would like to develop skills in customer service.
(ii) no more than three business owners would like to develop skills in customer service.
(b) Find the mean and variance of the binomial distribution that you have used in part (a).
(c) Using another binomial distribution, find the probability that at least four business owners did not say that they would like to develop skills in marketing.

9 A game is played with a fair 6 -sided die. A player throws this die, and if the result is $2,3,4$ or 5 , that result is the player's score. If the result of the player's throw is a 1 or 6 , the player throws a second time and the score is the sum of the two numbers from the two throws. Events $A$ and $B$ are defined as follows.
$A$ : The player's score is $5,6,7,8$ or 9 ,
$B$ : The player has two throws.
(a) Draw a tree diagram to represent this situation, showing all possible outcomes.
(b) Show that $\mathrm{P}(A)=\frac{1}{3}$.
(c) Find the probability that a player gets a score of 5, 6, 7, 8 or 9 in two throws.
(d) Describe what is meant by the probability $\mathrm{P}(A \mid B)$ in this context and find its value.

10 (a)


Eight pairs of values of variables $x$ and $y$ are measured. The sketches above show possible scatter diagrams for the data.
(i) State which of the sketches, A or B , could represent a product moment correlation coefficient of approximately -0.9 , justifying your answer.
(ii) Suggest an approximate value for the product moment correlation coefficient for the remaining sketch A or B .
(b) The monthly earnings, $y$ thousand dollars, of 8 employees of different ages, $x$ years, in a particular company are given in the following table.

| Employee | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | 18 | 20 | 22 | 27 | 35 | 45 | 55 | 60 |
| $y$ | 2.75 | 2.85 | 3.05 | 3.25 | 4.96 | 5.85 | 6.85 | 6.92 |

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator.
(ii) Find the product moment correlation coefficient and comment on its value in the context of the data.
(iii) Find the equation of the regression line of $y$ on $x$, giving your answer in the form $y=a x+b$, giving the values of $a$ and $b$ correct to 3 significant figures. Sketch this line on your scatter diagram.
(iv) Use the equation of your regression line to calculate an estimate for the monthly earnings of a 40 -year old employee in the company. Comment on the reliability of your estimate.

11 A scientist claims that in a particular forest, the mean height of mature red pine trees is greater than 27.8 m . To test this, she takes a random sample of 60 of these trees and measures their heights, $x \mathrm{~m}$. These heights are summarised as follows.

$$
\sum x=1698 \quad \sum x^{2}=48650
$$

(a) Test, at the $5 \%$ significance level, whether the scientist's claim is supported by the data.

The heights of a new random sample of 80 mature red pine trees from a different forest are measured. The mean height of the trees in this sample is $k \mathrm{~m}$ and the population variance is assumed to be $9.4 \mathrm{~m}^{2}$. A test, at the $10 \%$ significance level, fails to reject the belief that the population mean height of mature red pine trees is 27.8 m .
(b) Find the set of possible values of $k$.

12 The masses, in kg, of pears and apples sold by a supermarket have independent normal distributions with means and standard deviations as shown in the following table.

|  | Mean | Standard deviation |
| :--- | :---: | :---: |
| Pears | 0.19 | 0.028 |
| Apples | 0.12 | 0.016 |

(a) Find the probability that the mass of a randomly chosen pear is less than 0.21 kg .
(b) Find the probability that two randomly chosen pears each have a mass of more than 0.21 kg .
(c) Without any calculation, explain why the probability that the total mass of two randomly chosen pears is more than 0.42 kg is greater than your answer to part (b).
(d) Find the probability that a randomly chosen pear has a mass greater than the mass of a randomly chosen apple.

Mrs Sung is making a fruit salad. She randomly chooses 6 pears and 4 apples at the supermarket.
(e) Find the probability that the total mass of the 6 pears and 4 apples is less than 1.7 kg .

Pears cost $\$ 10$ per kg and apples cost $\$ 12$ per kg.
(f) Find the probability that Mrs Sung's 6 pears and 4 apples cost between $\$ 16$ and $\$ 18$.

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