Mathematics

Singapore-Cambridge General Certificate of Education Advanced Level Higher 1

(Syllabus 8865)

(Updated for examination from 2021)

CONTENTS

Significant changes to the syllabus are indicated by black vertical lines either side of the text.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

PREAMBLE

The applications of mathematics extend beyond the sciences and engineering domains. A basic understanding of mathematics and statistics, and the ability to think mathematically and statistically are essential for an educated and informed citizenry. For example, social scientists use mathematics to analyse data, support decision making, model behaviour, and study social phenomena.

H1 Mathematics provides students with a foundation in mathematics and statistics that will support their business or social sciences studies at the university. It is particularly appropriate for students without O-Level Additional Mathematics because it offers an opportunity for them to learn important mathematical concepts and skills in algebra and calculus that were taught in Additional Mathematics. Students will also learn basic statistical methods that are necessary for studies in business and social sciences.

SYLLABUS AIMS

The aims of H1 Mathematics are to enable students to:

(a) acquire mathematical concepts and skills to support their tertiary studies in business and the social sciences

(b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving

(c) connect ideas within mathematics and apply mathematics in the context of business and social sciences

(d) experience and appreciate the value of mathematics in life and other disciplines.

ASSESSMENT OBJECTIVES (AO)

There are three levels of assessment objectives for the examination.

The assessment will test candidates' abilities to:

AO1 Understand and apply mathematical concepts and skills in a variety of problems, including those that may be set in unfamiliar contexts, or require integration of concepts and skills from more than one topic.

AO2 Formulate real-world problems mathematically, solve the mathematical problems, interpret and evaluate the mathematical solutions in the context of the problems.

AO3 Reason and communicate mathematically through making deductions and writing mathematical explanations and arguments.

USE OF A GRAPHING CALCULATOR (GC)

The use of an approved GC without computer algebra system will be expected. The examination papers will be set with the assumption that candidates will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, candidates are required to present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, candidates should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Students should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

LIST OF FORMULAE AND STATISTICAL TABLES

Candidates will be provided in the examination with a list of formulae and statistical tables.

INTEGRATION AND APPLICATION

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

Possible list of H1 Mathematics applications and contexts:

Applications and contexts	**Some possible topics involved**
Optimisation problems (e.g. maximising profits, minimising costs)	Inequalities; System of linear equations; Calculus
Population growth, radioactive decay	Exponential and logarithmic functions
Financial maths (e.g. profit and cost analysis, demand and supply, banking, insurance)	Equations and inequalities; Probability; Sampling distributions; Correlation and regression
Games of chance, elections	Probability
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression

The list illustrates some types of contexts in which the mathematics learnt in the syllabus may be applied, and is by no means exhaustive. While problems may be set based on these contexts, no assumptions will be made about the knowledge of these contexts. All information will be self-contained within the problem.

SCHEME OF EXAMINATION PAPERS

For the examination in H1 Mathematics, there will be one 3-hour paper marked out of 100 as follows:

Section A (Pure Mathematics – 40 marks) will consist of about 5 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

Section B (Probability and Statistics – 60 marks) will consist of 6 to 8 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be at least two questions, with at least one in each section, on the application of Mathematics in real-world contexts, including those from business and the social sciences. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer **all** questions.

CONTENT OUTLINE

	Topics/Sub-topics	Content
SECTION A: PURE MATHEMATICS		
1	**Functions and Graphs**	
1.1	Exponential and logarithmic functions and Graphing techniques	Include: • concept of function as a rule or relationship where for every input there is only one output • use of notations such as $\mathrm{f}(x) = x^2 + 5$ • functions e^x and $\ln x$ and their graphs • exponential growth and decay • logarithmic growth • equivalence of $y = \mathrm{e}^x$ and $x = \ln y$ • laws of logarithms • use of a graphing calculator to graph a given function • characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes (horizontal and vertical) Exclude: • use of the terms domain and range • use of notation $\mathrm{f} : x \mapsto x^2 + 5$ • change of base of logarithms

	Topics/Sub-topics	Content
1.2	Equations and inequalities	Include: • conditions for a quadratic equation to have (i) two real roots, (ii) two equal roots, and (iii) no real roots • conditions for $ax^2 + bx + c$ to be always positive (or always negative) • solving simultaneous equations, one linear and one quadratic, by substitution • solving quadratic equations and inequalities in one unknown analytically • solving inequalities by graphical methods • formulating an equation or a system of linear equations from a problem situation • finding the approximate solution of an equation or a system of linear equations using a graphing calculator
2	**Calculus**	
2.1	Differentiation	Include: • derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a point • use of standard notations $\mathrm{f}'(x)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$ • derivatives of x^n for any rational n, e^x, $\ln x$, together with constant multiples, sums and differences • use of chain rule • graphical interpretation of $\mathrm{f}'(x) > 0$, $\mathrm{f}'(x) = 0$ and $\mathrm{f}'(x) < 0$ • use of the first derivative test to determine the nature of the stationary points (local maximum and minimum points and points of inflexion) in simple cases • locating maximum and minimum points using a graphing calculator • finding the approximate value of a derivative at a given point using a graphing calculator • finding equations of tangents to curves • local maxima and minima problems • connected rates of change problems Exclude: • differentiation from first principles • derivatives of products and quotients of functions • use of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$ • differentiation of functions defined implicitly or parametrically • finding non-stationary points of inflexion • relating the graph of $y = \mathrm{f}'(x)$ to the graph of $y = \mathrm{f}(x)$

	Topics/Sub-topics	Content
2.2	Integration	Include: • integration as the reverse of differentiation • integration of x^n for any rational n, and e^x, together with constant multiples, sums and differences • integration of $(ax + b)^n$ for any rational n, and $e^{(ax + b)}$ • definite integral as the area under a curve • evaluation of definite integrals • finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves • finding the approximate value of a definite integral using a graphing calculator Exclude: • definite integral as a limit of sum • approximation of area under a curve using the trapezium rule • area below the x-axis
SECTION B: PROBABILITY AND STATISTICS		
3	**Probability and Statistics**	
3.1	Probability	Include: • addition and multiplication principles for counting • concepts of permutation (^nP_r) and combination (^nC_r) • arrangements of distinct objects in a line including cases involving restriction • addition and multiplication of probabilities • mutually exclusive events and independent events • use of tables of outcomes, Venn diagrams, tree diagrams, and permutations and combinations techniques to calculate probabilities • calculation of conditional probabilities in simple cases • use of: $\mathrm{P}(A') = 1 - \mathrm{P}(A)$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

	Topics/Sub-topics	**Content**
3.2	Binomial distribution	Include: • knowledge of the binomial expansion of $(a + b)^n$ for positive integer n • binomial random variable as an example of a discrete random variable • concept of binomial distribution $B(n, p)$ and use of $B(n, p)$ as a probability model, including conditions under which the binomial distribution is a suitable model • use of mean and variance of a binomial distribution (without proof)
3.3	Normal distribution	Include: • concept of a normal distribution as an example of a continuous probability model and its mean and variance; use of $N(\mu, \sigma^2)$ as a probability model • standard normal distribution • finding the value of $P(X < x_1)$ or a related probability given the values of x_1, μ, σ • symmetry of the normal curve and its properties • finding a relationship between x_1, μ, σ given the value of $P(X < x_1)$ or a related probability • solving problems involving the use of $E(aX + b)$ and $\text{Var}(aX + b)$ • solving problems involving the use of $E(aX + bY)$ and $\text{Var}(aX + bY)$, where X and Y are independent Exclude normal approximation to binomial distribution.
3.4	Sampling	Include: • concepts of population and simple random sample. • concept of the sample mean $\overline{X}$ as a random variable with $E(\overline{X}) = \mu$ and $\text{Var}(\overline{X}) = \frac{\sigma^2}{n}$ • distribution of sample means from a normal population • use of the Central Limit Theorem to treat sample mean as having normal distribution when the sample size is sufficiently large (e.g. $n \geqslant 30$) • calculation of unbiased estimates of the population mean and variance from a sample, including cases where the data are given in summarised form Σx and Σx^2, or $\Sigma(x - a)$ and $\Sigma(x - a)^2$

<table>
<tr><th></th><th>Topics/Sub-topics</th><th>Content</th></tr>
<tr><td>3.5</td><td>Hypothesis testing</td><td>Include:
concepts of null hypothesis (H_0) and alternative hypotheses (H_1), test statistic, critical region, critical value, level of significance and p-value
formulation of hypotheses and testing for a population mean based on:
a sample from a normal population of known variance
a large sample from any population
1-tail and 2-tail tests
interpretation of the results of a hypothesis test in the context of the problem
Exclude the use of the term ‘Type I’ error, concept of Type II error and testing the difference between two population means.</td></tr>
<tr><td>3.6</td><td>Correlation and Linear regression</td><td>Include:
use of scatter diagram to determine if there is a plausible linear relationship between the two variables
correlation coefficient as a measure of the fit of a linear model to the scatter diagram
finding and interpreting the product moment correlation coefficient (in particular, values close to –1, 0 and 1)
concepts of linear regression and method of least squares to find the equation of the regression line
concepts of interpolation and extrapolation
use of the appropriate regression line to make prediction or estimate a value in practical situations, including explaining how well the situation is modelled by the linear regression model
Exclude:
derivation of formulae
relationship $r^2 = b_1b_2$, where b_1 and b_2 are regression coefficients
hypothesis tests
use of a square, reciprocal or logarithmic transformation to achieve linearity</td></tr>
</table>

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n-tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\ngtr$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\nless$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$
	$\frac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	the function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx ; δx	an increment of x
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \, \mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}, \ddot{x}, \ldots$	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions

7. *Complex Numbers*

i	the square root of –1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\mathrm{Re}\, z$	the real part of z, $\mathrm{Re}\,(x + \mathrm{i}y) = x$
$\mathrm{Im}\, z$	the imaginary part of z, $\mathrm{Im}\,(x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{x^2 + y^2}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations, $x_1, x_2, \ldots$occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$of the discrete random variable X
$\mathrm{f}(x)$, $\mathrm{g}(x)\ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x)\ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binomial distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum\left(x - \bar{x}\right)^2$
φ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample