

Essentials of Modern Physics

Singapore-Cambridge General Certificate of Education
Advanced Level Higher 3 (2018)

(Syllabus 9811)

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INTRODUCTION

The syllabus has been designed to build on and extend the content coverage of the H2 Physics syllabus. Candidates should simultaneously offer H2 Physics.

AIMS

The aims of a course based on this syllabus should be to:

1. provide students who have exceptional ability and interest in Physics an introduction to some specialised advanced topics in Modern Physics and enable them to acquire good understanding and appreciation of these topics at an undergraduate level
2. develop abilities and skills that:
 - 2.1 are relevant to the study and practice of science
 - 2.2 are useful in everyday life
 - 2.3 encourage efficient and safe practice
 - 2.4 encourage effective communication.
3. develop attitudes relevant to science such as:
 - 3.1 concern for accuracy and precision
 - 3.2 objectivity
 - 3.3 integrity
 - 3.4 the skills of enquiry
 - 3.5 initiative
 - 3.6 inventiveness.
4. promote an awareness:
 - 4.1 that the study and practice of Physics are co-operative and cumulative activities, and are subject to social, economic, technological, ethical and cultural influences and limitations
 - 4.2 that the implications of Physics may be both beneficial and detrimental to the individual, the community and the environment
 - 4.3 of the importance of the use of IT for communications, as an aid to experiments and as a tool for the interpretation of results
 - 4.4 that Physics transcends national boundaries and that the language of science, correctly and rigorously applied, is universal.
5. stimulate students and create a sustained interest in Physics so that the study of the subject is enjoyable and satisfying.

ASSESSMENT OBJECTIVES

The assessment objectives listed below reflect those parts of the aims that will be assessed in the examination.

A Knowledge with understanding

Candidates should be able to demonstrate knowledge and understanding in relation to:

1. scientific phenomena, facts, laws, definitions, concepts, theories
2. scientific vocabulary, terminology, conventions (including symbols, quantities and units)
3. scientific instruments and apparatus, including techniques of operation and aspects of safety
4. scientific quantities and their determination
5. scientific and technological applications with their social, economic and environmental implications.

The syllabus content defines the factual knowledge that candidates may be required to recall and explain. Questions testing these objectives will often begin with one of the following words: *define, state, describe* or *explain*. (See the glossary of terms.)

B Handling, applying and evaluating information

Candidates should be able – in words or by using written, symbolic, graphical and numerical forms of presentation – to:

1. locate, select, organise and present information from a variety of sources
2. translate information from one form to another
3. manipulate numerical and other data
4. use information to identify patterns, report trends, draw inferences and report conclusions
5. present reasoned explanations for phenomena, patterns and relationships
6. make predictions and put forward hypotheses
7. apply knowledge, including principles, to novel situations
8. evaluate information and hypotheses
9. demonstrate an awareness of the limitations of physical theories and models.

These assessment objectives cannot be precisely specified in the syllabus content because questions testing such skills may be based on information that is unfamiliar to the candidate. In answering such questions, candidates are required to use principles and concepts that are within the syllabus and apply them in a logical, reasoned or deductive manner to a novel situation. Questions testing these objectives will often begin with one of the following words: *predict, suggest, deduce, calculate* or *determine*. (See the glossary of terms.)

SCHEME OF ASSESSMENT

There is only one paper for this subject and it is of 3 hours duration. There will be two sections within the paper.

Section A will consist of a variable number of compulsory structured questions. The last of these will be a longer, data-response or comprehension question carrying approximately 20 marks. This section will carry a total of 60 marks.

Section B will consist of four longer structured questions of which candidates will answer two. Each question will carry 20 marks.

MARKS ALLOCATED TO LEARNING ASSESSMENT OBJECTIVES

<i>Knowledge with understanding</i>	(Assessment Objectives A1–A5)	40%
<i>Handling, applying and evaluating information</i>	(Assessment Objectives B1–B9)	60%

ADDITIONAL INFORMATION

Mathematical Requirements

The mathematical requirements are given on pages 10 and 11.

Data and Formulae

Data and Formulae, as printed on page 15 and 16, will appear as pages 2 and 3 in the examination paper.

Symbols, Signs and Abbreviations

Wherever symbols, signs and abbreviations are used in examination papers, the recommendation made in the ASE publication *SI Units, Signs, Symbols and Abbreviations* (1981) will be followed, except where these have been superseded by *Signs, Symbols and Systematics (The ASE Companion to 16–19 Science, 2000)*. The units kWh, atmosphere, eV and unified atomic mass unit (u) may be used in examination papers without further explanation.

SUBJECT CONTENT

1. Relativity

Content

- Inertial frames of reference
- Galilean transformation equations
- Postulates of special relativity
- Lorentz transformation equations
- Consequences of special relativity

Learning Outcomes

Candidates should be able to:

- (a) show understanding of what is meant by a frame of reference
- (b) recall and use Galilean transformation equations between frames of reference
- (c) use Galilean transformation equations to calculate relative velocities
- (d) state the failure of Galilean transformation equations when applied to a moving source of light
- (e) discuss qualitatively the results of the Michelson-Morley experiment and its implications on the ether theory. (Details of the experiment are not required.)
- (f) show understanding of what is meant by an inertial frame of reference
- (g) state the postulates of the special theory of relativity
- (h) discuss the concept of simultaneity
- (i) show understanding of the terms proper time and proper length
- (j) derive the time dilation formula and length contraction formula, making use of the Lorentz factor
- (k) apply the time dilation formula and the length contraction formula in related situations or to solve problems
- (l) apply the Lorentz transformation equations to solve one-dimensional problems
- (m) show an understanding of rest mass
- (n) distinguish between rest mass energy and total energy and solve related problems
- (o) use the mass-energy equivalence relationship in related situations or to solve problems
- (p) solve problems involving relativistic momentum and energy.

2. Quantum Theory of Light

Content

- Blackbody radiation
- Planck's hypothesis
- Compton effect
- Energy levels in atoms
- Laser

Learning Outcomes

Candidates should be able to:

- (a) show an understanding of what is meant by ideal blackbody radiation
- (b) apply Wien's displacement law in related situations or to solve problems
- (c) discuss qualitatively the failure of classical theory to explain blackbody radiation at high frequencies
- (d) describe qualitatively Planck's hypothesis of blackbody radiation
- (e) derive, using relativistic mechanics, the equation $p = h/\lambda$ for a photon and use the expression in related situations or to solve problems
- (f) show an understanding of the Compton shift effect and how it supports the concept of the photon
- (g) derive, using relativistic mechanics, the Compton shift equation and use the equation in related situations or to solve problems
- (h) show an understanding of discrete electron energy levels in isolated atoms (Bohr theory) and use the terms absorption, spontaneous emission and stimulated emission to describe photon processes involving electron transitions between energy levels, including the typical lifetime of such transitions
- (i) recall and use the equation $hf = E_1 - E_2$ to solve problems
- (j) apply the relation $N_x = N_0 \exp(-(E_x - E_0)/kT)$ to explain and solve problems on the population distribution of atoms with energy E_x
- (k) describe the process of population inversion and explain why this cannot be achieved with just two energy levels
- (l) use population inversion and stimulated emission to explain the action of a laser, using the He-Ne laser as a specific example. (Details on the structure and operation of the laser are not required.)

3. Matter Waves

Content

- Wave-particle duality
- Wave function
- X-ray and electron diffraction

Learning Outcomes

Candidates should be able to:

- show a qualitative understanding of the concept of wave-particle duality and the principle of complementarity
- show an understanding that a particle can be described using a wave function Ψ and give a simple mathematical form of the free particle wave function
- discuss qualitatively the probabilistic interpretation of the wave function and state that the square of the wave function amplitude $|\Psi|^2$ gives the probability density
- show an understanding of the normalisation of a wave function
- show an understanding that the resolving power of optical instruments is determined by the wavelength of the radiation and use this to explain how electron microscopes can achieve higher resolution than normal optical microscopes
- describe the use of X-ray diffraction to probe crystal structures
- recall and apply Bragg's equation $n\lambda = 2d \sin\theta$ to solve problems
- extend the use of Bragg's equation to electron diffraction in probing the surfaces of solids. (Only non-relativistic cases are considered.)

4. Quantum Mechanics

Content

- Schrödinger equation and solutions
- Quantisation of energy
- Barrier tunnelling
- Scanning tunnelling microscope

Learning Outcomes

Candidates should be able to:

- show an understanding that the Schrödinger equation can be used to explain the behaviour of atomic particles
- process a given wave function in the following ways:
 - verify that it is a solution to the time-independent Schrödinger equation; and
 - use the normalisation condition $\int_{-\infty}^{\infty} |\Psi|^2 dx = 1$ to find the normalisation constant for the wave function

- (c) derive, using the de Broglie relation and the concept of stationary wave, the allowed energy states for a particle trapped in a box of length L to be $(n^2h^2)/(8mL^2)$
- (d) apply the equation for the energy states of a particle trapped in a box to solve problems
- (e) describe the energy levels of a harmonic oscillator and deduce that there is a zero-point energy
- (f) explain the phenomenon of quantum tunnelling across a potential barrier by a wave function and compare this with the classical approach
- (g) sketch the wave function tunnelling across a potential barrier
- (h) apply the concept of quantum tunnelling to explain the radioactive decay of atomic nuclei with α emission.

5. Solid State Physics

Content

- Bonding in solids
- Free electron theory of metals
- Band theory of solids
- Semi-conductor devices
- Hall effect

Learning Outcomes

Candidates should be able to:

- (a) discuss the bonding in ionic solids (including metallic solids) and covalent solids with reference to electron spins
- (b) explain the bonding in molecules using potential-energy diagrams
- (c) describe crystal structures including simple cubic, face-centred cubic (fcc), body-centred cubic (bcc) and hexagonal close-packed (hcp)
- (d) solve problems on equilibrium separation and dissociation energy of atoms in crystals
- (e) apply the Drude model of electrical conduction to calculate the drift velocity, current density and conductivity and discuss its failure to account for the temperature dependence of resistivity
- (f) show an understanding of the terms electron density, Fermi level and Fermi energy
- (g) calculate Fermi energy of metals from given data. (Recall of the equation is not required.)
- (h) show an understanding that electrons follow the Fermi-Dirac distribution, with reference to Fermi function and density of energy states. (Recall of the equations is not required.)
- (i) account for the differences in electrical conductivities for conductors, insulators and semiconductors using the concept of Fermi level and band theory
- (j) distinguish between n-type and p-type semiconductors
- (k) explain the Zener effect
- (l) describe the Hall effect in relation to the Hall voltage, Hall coefficient, types and densities of charge carriers

- (m) outline the use of the Hall probe
- (n) apply the Hall effect to solve problems, including the derivation of the Hall voltage.

6. Photonics

Content

- Refractive index
- Transmittance and reflectance
- Polarisation
- Optical fibres

Learning Outcomes

Candidates should be able to:

- (a) use the concept of refractive index to solve problems involving speed, wavelength and frequency of waves
- (b) recall and apply Snell's Law to solve multiple boundary problems and to discuss the phenomenon of total internal reflection
- (c) discuss qualitatively the phenomenon of dispersion of light in a medium
- (d) show an understanding of the concept of polarisation and how polarisers work
- (e) recall and apply Malus' Law, $I = I_0 \cos^2 \theta$, to calculate the intensity of light transmitted by a plane polariser
- (f) discuss and apply qualitatively the concepts of transmittance and reflectance of a light wave incident on a dielectric. (Fresnel relations and its derivation are not required.)
- (g) show an understanding of Brewster's angle and use it to explain how to produce plane polarised light
- (h) calculate Brewster's angle using the relation, $\tan \theta_B = n_2/n_1$ and to solve related problems
- (i) distinguish qualitatively between plane and circularly polarised light, including how the electric field and magnetic field vectors change with space and time
- (j) describe how plane and circularly polarised light can be obtained using the principle of birefringence
- (k) apply the equation $\frac{\delta}{2\pi} = \frac{d}{\lambda} \Delta n$, where Δn is the measure of birefringence, to solve problems involving circularly polarised light
- (l) show an understanding of light attenuation and use the relation, $I = I_0 \exp(-\mu x)$ in related situations and to solve problems
- (m) discuss the advantages and disadvantages of various types of optical fibres (e.g. step index, continuous index)
- (n) discuss the advantages and disadvantages between optical telecommunication and other forms (electrical wires, radiowaves, satellite networks etc.) of telecommunications, showing an understanding of the differences between single mode and multimode optical fibres.

MATHEMATICAL REQUIREMENTS

Arithmetic

Candidates should be able to:

- recognise and use expressions in decimal and standard form (scientific) notation
- use appropriate calculating aids (electronic calculator or tables) for addition, subtraction, multiplication and division. Find arithmetic means, powers (including reciprocals and square roots), sines, cosines, tangents (and the inverse functions), exponentials and logarithms (lg and ln)
- take account of accuracy in numerical work and handle calculations so that significant figures are neither lost unnecessarily nor carried beyond what is justified
- make approximate evaluations of numerical expressions (e.g. $\pi^2 = 10$) and use such approximations to check the magnitude of machine calculations.

Algebra

Candidates should be able to:

- change the subject of an equation. Most relevant equations involve only the simpler operations but may include positive and negative indices and square roots
- solve simple algebraic equations. Most relevant equations are linear but some may involve inverse and inverse square relationships. Linear simultaneous equations and the use of the formula to obtain the solutions of quadratic equations are included
- substitute physical quantities into physical equations using consistent units and check the dimensional consistency of such equations
- formulate simple algebraic equations as mathematical models of physical situations, and identify inadequacies of such models
- recognise and use the logarithmic forms of expressions like ab , a/b , x^n , e^{kx} ; understand the use of logarithms in relation to quantities with values that range over several orders of magnitude
- manipulate and solve equations involving logarithmic and exponential functions
- perform differentiation and integration of simple functions, including trigonometric, exponential and logarithmic functions
- express small changes or errors as percentages and *vice versa*
- comprehend and use the symbols $<$, $>$, \leq , \geq , \ll , \gg , \approx , \propto , $\langle x \rangle$ ($= \bar{x}$), Σ , Δx , δx , $\sqrt{\quad}$.

Geometry and trigonometry

Candidates should be able to:

- calculate areas of right-angled and isosceles triangles, circumference and area of circles, areas and volumes of rectangular blocks, cylinders and spheres
- use Pythagoras' theorem, similarity of triangles, the angle sum of a triangle

- (c) use sines, cosines and tangents (especially for 0° , 30° , 45° , 60° , 90°). Use the trigonometric relationships for triangles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}; \quad a^2 = b^2 + c^2 - 2bc \cos A$$

- (d) use $\sin \theta \approx \tan \theta \approx \theta$ and $\cos \theta \approx 1$ for small θ ; $\sin^2 \theta + \cos^2 \theta = 1$
- (e) understand the relationship between degrees and radians (defined as arc/radius), translate from one to the other and use the appropriate system in context.

Vectors

Candidates should be able to:

- (a) find the resultant of two coplanar vectors, recognising situations where vector addition is appropriate
- (b) obtain expressions for components of a vector in perpendicular directions, recognising situations where vector resolution is appropriate.

Graphs

Candidates should be able to:

- (a) translate information between graphical, numerical, algebraic and verbal forms
- (b) select appropriate variables and scales for graph plotting
- (c) for linear graphs, determine the slope, intercept and intersection
- (d) choose, by inspection, a straight line which will serve as the line of best fit through a set of data points presented graphically
- (e) recall standard linear form $y = mx + c$ and rearrange relationships into linear form where appropriate
- (f) sketch and recognise the forms of plots of common simple expressions like $1/x$, x^2 , $1/x^2$, $\sin x$, $\cos x$, e^{-x}
- (g) use logarithmic plots to test exponential and power law variations
- (h) understand, draw and use the slope of a tangent to a curve as a means to obtain the gradient, and use notation in the form dy/dx for a rate of change
- (i) understand and use the area below a curve where the area has physical significance.

Any calculator used must be on the Singapore Examinations and Assessment Board list of approved calculators.

GLOSSARY OF TERMS USED IN PHYSICS PAPERS

It is hoped that the glossary will prove helpful to candidates as a guide, although it is not exhaustive. The glossary has been deliberately kept brief not only with respect to the number of terms included but also to the descriptions of their meanings. Candidates should appreciate that the meaning of a term must depend in part on its context. They should also note that the number of marks allocated for any part of a question is a guide to the depth of treatment required for the answer.

1. *Define (the term(s) ...)* is intended literally. Only a formal statement or equivalent paraphrase, such as the defining equation with symbols identified, being required.
2. *What is meant by ...* normally implies that a definition should be given, together with some relevant comment on the significance or context of the term(s) concerned, especially where two or more terms are included in the question. The amount of supplementary comment intended should be interpreted in the light of the indicated mark value.
3. *Explain* may imply reasoning or some reference to theory, depending on the context.
4. *State* implies a concise answer with little or no supporting argument, e.g. a numerical answer that can be obtained 'by inspection'.
5. *List* requires a number of points with no elaboration. Where a given number of points is specified, this should not be exceeded.
6. *Describe* requires candidates to state in words (using diagrams where appropriate) the main points of the topic. It is often used with reference either to particular phenomena or to particular experiments. In the former instance, the term usually implies that the answer should include reference to (visual) observations associated with the phenomena. The amount of description intended should be interpreted in the light of the indicated mark value.
7. *Discuss* requires candidates to give a critical account of the points involved in the topic.
8. *Deduce/Predict* implies that candidates are not expected to produce the required answer by recall but by making a logical connection between other pieces of information. Such information may be wholly given in the question or may depend on answers extracted in an earlier part of the question.
9. *Suggest* is used in two main contexts. It may either imply that there is no unique answer or that candidates are expected to apply their general knowledge to a 'novel' situation, one that formally may not be 'in the syllabus'.
10. *Calculate* is used when a numerical answer is required. In general, working should be shown.
11. *Measure* implies that the quantity concerned can be directly obtained from a suitable measuring instrument, e.g. length, using a rule, or angle, using a protractor.
12. *Determine* often implies that the quantity concerned cannot be measured directly but is obtained by calculation, substituting measured or known values of other quantities into a standard formula, e.g. the Young modulus, relative molecular mass.
13. *Show* is used when an algebraic deduction has to be made to prove a given equation. It is important that the terms being used by candidates are stated explicitly.
14. *Estimate* implies a reasoned order of magnitude statement or calculation of the quantity concerned. Candidates should make such simplifying assumptions as may be necessary about points of principle and about the values of quantities not otherwise included in the question.

15. *Sketch*, when applied to graph work, implies that the shape and/or position of the curve need only be qualitatively correct. However, candidates should be aware that, depending on the context, some quantitative aspects may be looked for, e.g. passing through the origin, having an intercept, asymptote or discontinuity at a particular value. On a sketch graph it is essential that candidates clearly indicate what is being plotted on each axis.
16. *Sketch*, when applied to diagrams, implies that a simple, freehand drawing is acceptable: nevertheless, care should be taken over proportions and the clear exposition of important details.
17. *Compare* requires candidates to provide both similarities and differences between things or concepts.

TEXTBOOKS

Teachers may find reference to the following books helpful.

Physics for Scientists and Engineers with Modern Physics (5th Edition), by R Serway, published by Saunders, ISBN 0-030-20974-9

Fundamentals of Physics (Extended – 6th Edition), by R Resnick, D Halliday and J Walker, published by Wiley, ISBN 0-471-22863-X

Physics: Principles with Applications (5th Edition), by D C Giancoli, published by Prentice Hall, ISBN 0-13611-971-9

Physics (5th Edition), by R Resnick, D Halliday and K Krane, published by Wiley, ISBN 0-471-32057-9

Contemporary College Physics (3rd Edition) by Jones and Childers, published by McGraw-Hill, ISBN 0-07241-512-6

Teachers are encouraged to choose texts for class use that they feel will be of interest to their students and will support their own teaching style.

SUMMARY OF KEY QUANTITIES, SYMBOLS AND UNITS

The following list illustrates the symbols and units that will be used in question papers.

Quantity	Usual symbols	Usual unit
<i>Base Quantities</i>		
mass	m	kg
length	l	m
time	t	s
electric current	I	A
thermodynamic temperature	T	K
amount of substance	n	mol
<i>Other Quantities</i>		
distance	d	m
displacement	s, x	m
area	A	m^2
volume	V, v	m^3
density	ρ	kg m^{-3}
speed	u, v, w, c	m s^{-1}
velocity	u, v, w, c	m s^{-1}
acceleration	a	m s^{-2}
acceleration of free fall	g	m s^{-2}
force	F	N
weight	W	N
momentum	p	N s
work	w, W	J
energy	E, U, W	J
potential energy	E_p	J
kinetic energy	E_k	J
power	P	W
period	T	s
frequency	f	Hz
angular frequency	ω	rad s^{-1}
wavelength	λ	m
speed of electromagnetic waves	c	m s^{-1}
electric charge	Q	C
elementary charge	e	C
electric potential	V	V
electric potential difference	V	V
electromotive force	E	V
electric field strength	E	$\text{N C}^{-1}, \text{V m}^{-1}$
permittivity of free space	ϵ_0	F m^{-1}
magnetic flux	Φ	Wb
magnetic flux density	B	T
permeability of free space	μ_0	H m^{-1}
Boltzmann constant	k	J K^{-1}
Avogadro constant	N_A	mol^{-1}
number density (number per unit volume)	n	m^{-3}
Planck constant	h	J s
work function energy	ϕ	J
atomic mass	m_a	kg, u
electron mass	m_e	kg, u
neutron mass	m_n	kg, u
proton mass	m_p	kg, u

DATA AND FORMULAE

Data

speed of light in free space	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$ $(1/(36\pi)) \times 10^{-9} \text{ F m}^{-1}$
elementary charge	$e = 1.60 \times 10^{-19} \text{ C}$
the Planck constant	$h = 6.63 \times 10^{-34} \text{ Js}$
unified atomic mass constant	$u = 1.66 \times 10^{-27} \text{ kg}$
rest mass of electron	$m_e = 9.11 \times 10^{-31} \text{ kg}$
rest mass of proton	$m_p = 1.67 \times 10^{-27} \text{ kg}$
molar gas constant	$R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
the Avogadro constant	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
the Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
gravitational constant	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
acceleration of free fall	$g = 9.81 \text{ m s}^{-2}$

Formulae

Lorentz factor	$\gamma = (1 - (v/c)^2)^{-1/2}$
length contraction	$L = L_0 / \gamma$
time dilation	$T = \gamma T_0$
Lorentz transformation equations (1 dimension)	$x' = \gamma(x - vt)$ $t' = \gamma(t - vx/c^2)$
mass-energy equivalence	$E = \gamma m_0 c^2$ $E^2 = (pc)^2 + (m_0 c^2)^2$
Wien's displacement law	$\lambda_p T = 2.898 \times 10^{-3} \text{ m K}$
Compton shift formula	$\Delta\lambda = \frac{h}{mc} (1 - \cos \theta)$
population distribution of atoms with energy E_x	$N_x = N_0 \exp(-(E_x - E_0)/kT)$
time-independent Schrödinger equation	$E\Psi = -\frac{\hbar^2}{2m} \left(\frac{d^2\Psi}{dx^2} \right) + U\Psi$
allowed energy states for a particle in a box	$E_n = (n^2 \hbar^2)/(8mL^2)$
normalised wave function for a particle in a box	$\Psi = (2/L)^{1/2} \sin(n\pi x/L)$
transmission coefficient	$T \propto \exp(-2kd)$
	where $k = \sqrt{\frac{8\pi^2 m(U - E)}{h^2}}$
Drude model of electrical resistivity	$\rho = \frac{2m_e \langle v \rangle}{ne^2 \lambda}$

Fermi energy for metals	$E_F = \frac{h^2}{8m} \left(\frac{3n}{\pi} \right)^{2/3}$
density of energy states for electrons in a metal	$\rho(E) = \frac{4\pi(2m)^{3/2}}{h^3} \sqrt{E}$
Fermi function	$f(E) = \frac{1}{1 + \exp((E - E_F)/kT)}$
refractive index	$n = v_1/v_2$
phase difference of circularly polarised light	$\frac{\delta}{2\pi} = \frac{d}{\lambda} \Delta n$
Brewster's angle	$\tan \theta_B = n_2/n_1$
attenuation of light intensity	$I = I_0 \exp(-\mu x)$